DOCUMENT RESUME

ED 289 357 FL 017 071

AUTHOR Kessler, Carolyn
TITLE Linking Mathematics and Second Language Teaching.
PUB DATE Apr 87
NOTE 31p.; Paper presented at the Annual Meeting of the Teachers of English to Speakers of Other Languages (21st, Miami Beach, FL, April 21-27, 1987).
PUB TYPE Information Analyses (070) -- Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Cognitive Processes; *Communicative Competence (Languages); *English (Second Language); *English for Special Purposes; Limited English Speaking; Mathematical Concepts; *Mathematics Instruction; *Problem Solving; Second Language Learning
IDENTIFIERS *Content Area Teaching

ABSTRACT

A discussion of the relationship between second language proficiency and mathematics performance argues that the learning of a second language and mathematics can be effectively integrated. The conditions conducive to second language acquisition in mathematics contexts, the communicative competence needed for mathematics, and the link between mathematical thinking and performance in a second language are examined. Preliminary findings from research studies relating mathematics to second language acquisition in school settings, particularly among limited-English-proficient students, are examined and the implications for teaching are discussed. The processes of language learning and mathematical problem-solving share many cognitive and affective features. Language learning in this context can be enhanced by mathematics instruction that provides alternate ways of getting input through personalized and cooperative learning situations. It can also contribute to the development of literacy. (MSE)

**********
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
**********

ED289357

# LINKING MATHEMATICS AND SECOND LANGUAGE TEACHING

Carolyn Kessler
University of Texas at San Antonio

Paper presented at the 21st Annual Convention of Teachers of English to Speakers of Other Languages, Miami Beach, Florida, April 21-27, 1987.

FL017071

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

C. Kessler

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.



BEST COPY AVAILABLE

# LINKING MATHEMATICS AND SECOND LANGUAGE TEACHING

Carolyn Kessler
University of Texas at San Antonio

> Understanding the language used in mathematics is the primary factor that affects my performance. Some terms such as **quotient, exponent, perimeter** are truly beyond my knowledge though all of them may represent very simple, basic concepts in mathematics. Without help from dictionary or other source, the mathematic word problem just can not enter my brain. ... In addition to terms, I feel difficulty in understanding descriptions that involve socio-cultural contexts such as credit or installment. **I try so hard to figure out what the problem talks about...Once I met problem in understanding, I simply rejected the mathematics itself. I had full confidence in my ability to solve the mathematics problem yet I could not do it because of insufficient language proficiency.**

A Chinese graduate student studying second language acquisition and content-based instruction wrote this introspective account of doing mathematics in English. It is a clear articulation of some of the difficulties mathematics poses for second language learners. The negative consequences on attitudes and performance from not knowing the language and the socio-cultural contexts of mathematics problems are all part of this assessment.

Although language proficiency certainly impacts mathematical performance, the thesis of this paper is that learning mathematics and a second language can be effectively integrated. It addresses in particular conditions for:

(1) second language acquisition in mathematics contexts

(2) communicative competence needed for mathematics

(3) mathematics thinking and performance in a second language.

The paper begins with some preliminary findings from studies relating mathematics and second language acquisition in school settings. It concludes with a summary of implications for teaching.

## PILOT STUDIES ON INTEGRATING MATHEMATICS AND LANGUAGE ACQUISITION

Two areas of research on integrating mathematics and language acquisition currently underway in schools with large Hispanic populations in San Antonio and South Texas include: (1) contrasts between English proficient and limited English proficient (LEP) students, elementary and secondary school levels, (2) effects on mathematics performance of writing-to-learn projects, grades K-6. These are pilot studies; findings are preliminary.

### Contrasts Between LEP and Non-LEP Students

Examining similarities and differences between native English speakers and LEP students in mathematical performance is a broad area of research which is needed if we are to understand issues in integrating language and mathematics content.

One area of contrast between L1 and L2 students is that of prior experience or real-world contexts needed to solve mathematical problems successfully. A procedure we have used is to ask both L1 and L2 students to circle words in math problems that they did not know. The assumption is that words circled might give insight into

gaps in prior knowledge or experience relevant to solving the problem.

A class of 22 Hispanic sixth-graders, evenly divided between non-LEP and beginning level LEP students, was asked to circle any words not understand in the following problem:

> A rollercoaster ride in an amusement park lasts only 5 minutes. During the first hour that Sam stayed at the amusement park, he took 7 rides. How many minutes did Sam spend on rollercoaster rides?

No words were circled by native English speakers. They understood all words and could visualize the situation. Of the LEP children, 100 % circled **amusement**, 73 % **rollercoaster**, and 45% the words **during, stayed, rides**, and **spend.** In other words, no LEP child fully understood the amusement park context for the problem.

In another sixth-grade class, 6 LEP students of intermediate level proficiency and 6 native English speakers were asked to carry out the same type of task for the following:

> Mrs. Bennett's kettle holds enough fruit to make 8 jars of jelly at a time. Last summer Mrs. Bennett made 112 jars of jelly. How many kettlefuls of fruit did she have to cook?

Again, no native speaker reported language difficulties. Of the LEP students, 67 % circled **kettle** and **kettlefuls**, 50 % **enough**, 33 % **fruit** and **Mrs. Bennett's. Amusement parks** and **kettlefuls of fruit** illustrate socio-cultural aspects of many word problems in mathematics texts.

In spite of language difficulties, 90% of the LEP students said they liked math. When asked what was the hardest part of math, 85% made comments about language. Of the non-LEP students, 75% claimed to liked math. Only 18% cited language as the hardest part. Their focus was on difficulty in various mathematical operations. From our preliminary findings, language is a major stumbling block in mathematics for the LEP student and appears to override any concerns with processes or calculations specific to mathematics.

**Writing To Learn**

As part of several writing-to-learn projects in urban and rural areas of South Texas, LEP and non-LEP children have engaged extensively in learning to write their own mathematics problems. Initially, a common response to writing problems is a blank paper. Over time, however, the pattern changes dramatically. Not only do all children engage in problem writing, they look forward to doing it. Standardized achievement scores indicate that as children participate extensively in writing activities their scores go up in both language and mathematics. Preliminary findings suggest that greatest gains are made by LEP students.

Part of our data analysis focuses on case studies. Larry, like the others in his class of 22 bilingual Spanish/English migrant children in rural South Texas, entered fifth grade functionally illiterate in English. After extensive experiences in writing - dialogue journals,

class publications, among other activities - and participation in a classroom in which a sensitive teacher set a very positive affective climate, Larry quite literally wrote his way to reading. The other LEP children did too. By the end of March that year, Larry, who could hardly read or write in September, wrote the following word problem:

**Jaime went to the store**

Jaime had $7.30 and he bought a six pack of beer for Mr. B it cost $3.00 and a magnifying glass for $2.50 and a bag of chips for 50 c. How much did he spend? $6.00. How much did he have left? $1.30.

In addition to the necessary computations, this paper also includes a drawing of Jaime's going to the local convenience store. Mathematics had connection to Larry's world. It made sense and become a subject of interest, along with the rewards of learning to read and write. Success for Larry and his classmates in large measure resulted from affective changes in attitudes about school and from great quantities of language input provided through teacher and peer interactions in contexts that made reading and writing interesting and relevant. Larry's successful development of literacy and mathematical performance in English, his second language, illustrates in a basic, concrete way the interrelation of language and mathematics (Kessler, Quinn, Hayes 1985; Hayes, Bahruth & Kessler, in press).

Anna is from Mexico where she had five years of education in Spanish before entering a middle school in San Antonio, Texas. Anna is a good student who likes mathematics. She is succeeding at school even though her scores on the Stanford Achievement Test after one year in the U.S. show her reading comprehension in the 1st percentile and overall language proficiency in the 4th percentile. Her overall mathematics proficiency is in the 42nd percentile, however, with the highest score in mathematics calculations, the lowest in applications. After 12 months of schooling in English, Anna wrote this problem in her ESL class in a writing-to-learn activity:

> Mrs. Fleming have $100 in she go to the mall and she take 3 pairs of shos the cost of one pair is $20 and the other one is $15 and the other one is $20 How much Mrs. Fleming have to pay? and How many mony Mrs. Fleming have?
>
> cost of pair of choses is
> $20
> $20
> $15
> Total is $55
>
> $100
> -$55
> 45 Total

Anna designed a mathematics problem, expressed it meaningfully, and solved it accurately. And she liked doing it. She even used her teacher as the subject of her problem.

Explaining how writing mathematics problems can facilitate development both in English and in mathematics requires looking closely at factors in second language acquisition and mathematical reasoning as well as language competence needed for mathematics.

Explanations of Larry's and Anna's successes leads us to focus on these factors. Problems arise when links between form and meaning do not occur. Differences between L1 and L2 students in mathematics problem solving are probably more quantitative than qualitative in terms of abilility to link language form with understanding.

## SECOND LANGUAGE ACQUISITION IN MATHEMATICS CONTEXTS

Second language acquisition is a socio-cognitive process resulting from efforts to participate in communicative interactions. As the exchange and negotiation of conceptual, sociocultural, and affective content, mathematics provides a form of social and cognitive interaction that meets requirements for genuine language use. Mathematics provides a context in which L2 acquisition can occur. Larry's and Anna's efforts to write mathematics problems, for example, gave opportunities to use their newly developing language in a communicative context.

Current L2 acquisition theory assigns a critical role for the successful development of ESL to the school content areas. Mohan (1986) presents a comprehensive overview of relating language and content. Defining language acquisition as a natural, subconscious process that takes place when the focus is on communication or meaning, Krashen (1985) argues that this process is central to L2 development. Causative variables are the amount and quality of comprehensible input that the student receives and the set of

affective variables that alter the learner's openness to language input.

**Comprehensible Input**

When taught in ways that utilize visuo-spatial imagery, concrete objects, and other devices to clarify meaning, mathematics can provide a source of meaningful, relevant input for second language acquisition. Hands-on, concrete experiences such as used in taking measurements or in working with shapes can contribute to understanding meanings. So can graphs, charts, diagrams. The school's way of structuring the input and the learner's own set of characteristics brought to the L2 task interact to facilitate or inhibit the process.

**Affective Filter**

Anna likes mathematics. Her positive attitude helped her integrate her mathematics interests with her efforts to use English. Larry, to his surprise after years of failure, discovered that mathematics is relevant in his world and that he can be successful. He learned to like writing and solving problems. Larry's and Ann's teachers encouraged writing mathematics problems that drew on real-life experiences. Teacher-sensitivity to student language and mathematics proficiency contributes to setting the positive conditions needed for learning language and mathematics concurrently.

With effective instruction, mathematics can contribute to providing the necessary affective conditions that support language development. The interaction of affective and cognitive experiences is central to the development of a second language just as it is to the development of mathematical inquiry. The curiosity, surprise, or need to resolve a cognitive conflict that affectively drives mathematical thinking also drives language acquisition. In mathematics instruction, the teacher can tap the student's prior knowledge of the subject, build on it, and at the same time lead the student to a realization of that knowledge in a new language system. Affective variables may exert stronger effects with increasing age and maturity. The learner's attitudes towards the new language and culture and the content area of mathematics can interact positively or negatively. In designing her mathematics problem, Anna wrote about her ESL teacher and an imaginary trip to a shopping mall. Anyone familiar with the sub-culture of young adolescents in San Antonio is aware of how central shopping malls are to their way of life. Larry, too, wrote about his teacher and about shopping at the local convenience store, an important part of Larry's rural culture. Being encouraged to use mathematical thinking in interesting and relevant contexts is very possibly a factor contributing to Larry's and Anna's success in writing their problems.

## COMMUNICATIVE COMPETENCE AND MATHEMATICS

When children learn mathematics they learn the language needed to express meaning appropriate to the communication of mathematical ideas and they learn about the system of symbols that represent numbers, operations and the rules for manipulating them. Much of school mathematics focuses on representing ideas with symbols and manipulating those symbols according to specific rules. However, for understanding mathematics, students must be able to link those forms with meaning. This process of linking is at the heart of learning mathematics (Hiebert 1984).

Halliday (1975) defined a mathematics register as mathematical meanings with the terms or vocabulary used in expressing these ideas and the structure or sentences in which these terms appear. Dale & Cuevas 1987, Crandall, Dale, Rhodes & Spanos 1985; Kessler, Quinn & Hayes 1985; Cuevas 1984; Dawe 1984 have followed Halliday's characterization of the register to expand and refine it, particularly in terms of the linguistic difficulties that the language of mathematics poses for ESL learners. (For an extensive treatment, see Dale & Cuevas 1987).

In a broader sense, the language of mathematics may be viewed as communicative competence specific to mathematics, a subcomponent of cognitive/academic language proficiency (Cummins 1984). The mathematics register may be viewed as intersecting with an underlying cognitive analytic mathematics proficiency as defined by

Dawe (1984). The language of mathematics is a written, context-reduced language requiring cognitively demanding tasks.

The theoretical framework for communicative competence outlined by Canale (1983) is useful in analyzing language aspects of communication in mathematics. It identifies four components: grammatical, discourse, sociolinguistic, and strategic. Each component enters into processes linking language acquisition and mathematical reasoning.

**Grammatical Competence**

Grammatical or linguistic competence is that component which refers to the formal features of language, the language code. In addition to phonology, it includes vocabulary and syntax, features particularly relevant to the mathematics register.

Lexicon

Vocabulary knowledge for mathematical tasks has at least four distinct dimensions: (1) specific terms unique to mathematics, (2) the labels attached to mathematical symbols, (3) lexical items marked with unique meaning for the mathematics register, (4) collocations of words to express mathematical notions.

An example of difficulties with specific terms is seen in the exchange between an ESL learner and teacher on the meaning of the term **quotient**:

Student: What's this q-u-o-t-i-e-n-t? (spells the word)

Teacher: Math term for an answer in division.

Student: Thought I knew what to do, but when I saw that quotient,

I just stopped. Guess I ran out of English.

Learning to relate terms and symbols presents its own set of difficulties as in **X** means **multiplied by** or **times.**

The unique meanings of ordinary words in a mathematics context also present difficulties. As an example, a third grader followed the directions **Make a table** as used in mathematics by drawing a picture of a table as used in the home. Another could not add new meaning to the word **second**. For this student **second** had only one meaning, **two**. In an activity on **time**, the following exchange took place between the teacher and LEP student:

Teacher: What is the blue thing going around the clock.

Student: (no response)

Teacher: the second hand (giving the expected answer)

Student: (puzzled) clock have two hand

The teacher was presenting **the blue thing** as showing a unit of time. The student could not process the different meanings of **second**. In

another class of 10, no student knew the word **times in its meaning of multiplied by.**

Complex strings of words such as **lowest common demonimator** or lexical collocations as **square feet** present difficulty. A grade 4 LEP student could not resolve semantic conflicts between the notions of **square** and **feet.** Consequently, he completely missed the mathematical notion of **square feet.**

As Richards (1985) outlines, lexical knowledge implies knowing the constraints of function and situation on word choice such as the use of **table** or **square feet** in mathematics. It also includes knowing the network of word associations such as that between **product** and **multiplication.** Also, knowing a base form and its derivations makes it possible to relate sets of words such as **divide, division, divisor, dividend.**

Analyzing factors affecting academic achievement in a second language, Saville-Troike (1984) found that vocabulary is the most important linguistic component when learning academic content through a second language. Although many vocabulary problems are not unique to ESL students, they present perhaps quantitatively the most serious barriers to mathematics achievement for LEP students.

Syntax

Syntax presents at least three major sources of difficulty: (1) word-symbol correspondence for mathematical statements, (2) syntactic structures typically utilized in setting up mathematical problems, (3) logical connectors used in linking propositions.

An illustration of word-symbol correspondence is seen in an adult ESL student's interpretation of **5 x 4 = 20** as **five time four to twenty.** Inappropriate use of **to** in this expression could potentially confuse multiplication statements with ratio statements utilizing the preposition **to.**

Syntax interfaces with semantics in the structure of mathematics questions. On a grade 2 subtraction worksheet, questions guiding this task varied from **How many were there in all?, How many were there then?, How many more were there?, How many were left?, How many fewer were there?.** Syntactic variation on a single semantic notion can present considerable difficulties for the L2 learner.

Logical connectors are syntactic devices used to mark relationships between two or more propositions. Words or phrases such as **and, or, if...then, because, consequently,** serve a semantic, cohesive function indicating the nature of the relationship between the parts of a text. Mathematics texts typically include signals for addition or similarity, contradition, cause/effect or reason/result, and chronological or logical sequence (Celce-Murcia and Larsen-Freeman 1983). Research evidence (Dawe 1983) indicates that understanding

logical connectors is critical to the development of mathematical reasoning and function as a major factor in the failure of LEP students to solve problems successfully.

**Sociolinguistic Competence**

Sociolinguistic competence addresses appropriate use of language. As Canale (1983) explains, sociolinguistic competence refers to the degree to which the expression of particular language functons, attitudes and ideas is judged to be acceptable for a specific situation. Success in doing mathematics requires knowing how to use the mathematics register appropriately.

**How much ya got left?** vs. **How much money do you have?** may ask for the same information, but send different meanings from a sociolinguistic view. Development of the necessary language forms to carry out specific functions in a variety of ways and an awareness of the socio-mathematical significance of specific forms is a complex process closely paralleling cognitive growth and increased proficiency in the L2 system (Kessler, Quinn, & Hayes 1985).

**Discourse Competence**

Discourse competence deals with the connection of a series of utterances or sentences to form a meaningful entity. The connections that make a meaningful whole are achieved through cohesion in form and coherence in meaning. Cohesion deals with

overt markers linking propositions, such as logical connectors or anaphoric devices, which signal previous reference. Coherence, in contrast to cohesion, is often achieved without any overt connections. It draws on general knowledge of the real world and familiarity with specific contexts.

In terms of coherence, the lack of redundancy or paraphrase in math texts may create difficulty in linking form and function. To illustrate discourse difficulties, a class of grade 4 LEP children was asked to do a multiplication problem using symbols only. The class scored 51% accuracy. But when the same problem was given as a word problem, the accuracy level dropped to 6%.

Mathematics discourse involves understanding language functions such as the representative function, or telling how things are, and the directive function, or requesting others to do things (Searle 1976). Word problems require connecting lexical and syntactic forms with appropriate interpretations of language functions. A class of second graders, for example, had difficulty interpreting the following story problem:

> Sue has 32 cards.
> She gave Jim 15 cards.
> How many cards did Sue have?

The majority answered 15 rather than the correct response 17. Students may not have recognized the representative function expressed in possessing 32 cards and giving 15 away or in the

directive function requesting statement of the number of cards remaining.

**Strategic Competence**

Strategic competence is characterized by the strategies used in compensating for breakdowns in communication. When L2 learners do not understand a mathematics problem, they may make use of devices that allow them to try to solve the problem.

Hector, a nine-year-old intermediate level LEP student had difficulty understanding the problem:

> Ron saw a dog show. There were 7 small dogs and 2 large dogs. How many dogs were there in all?

Hector circled **show** and **large** as words he did not know. From the context he eventually understood **dog show.** Further re-reading with a long pause at **small** appeared to unlock the meaning of **large.** He explained to his teacher, however, that the expression **in all** helped most in solving the problem correctly. Hector used strategic competence to find appropriate words, in this case **in all**, to guide problem solving.

Appropriate strategic competence can be lacking as it was for Jose, a fourth-grader, in solving the following problem:

> There were 8 black cats. There were 6 white cats. How many more black cats were there?

In answering 14, Jose explained that **more** means **add**. His strategy of searching for key words like **more** led to missing the subtraction task.

Communicative competence in mathematics, as for other domains of language use, results from complex interactions of grammatical, sociolinguistic, discourse, and strategic competence. The nature of this interaction in any context is still very much open to research.

## MATHEMATICAL PROBLEM SOLVING IN SECOND LANGUAGE CONTEXTS

Although the precise role of language in learning mathematics is not yet clear, we may nevertheless consider that at a deeper cognitive level language works as a mediator in fundamental processes for doing mathematics and thinking mathematically. Contemporary researchers in the field of mathematics education fully recognize that there is much more to mathematics than mastering subject matter. Exploration of mathematical processes, strategies, and understandings that constitute mathematical thinking and performance are of primary interest. As part of this research effort, problem solving has emerged as one of the foremost topics of interest in mathematics education. Problem solving involves building a representation of the words of the problem and finding the solution of the problem using the rules of mathematics (Mayer 1986).

As a matter of definition, mathematical thinking may be distinguished from the body of knowledge - content and techniques - traditionally labelled as mathematics. Taking into account both cognitive and metacognitive considerations, Schoenfeld (1985) treats four major categories for investigating mathematical thinking: (1) cognitive resources, (2) heuristics, (3) control processes, (4) belief systems. We will examine each of these categories in the context of the LEP student and mathematics.

**Cognitive Resources**

Cognitive resources incorporate the knowledge of facts, concepts, and algorithms an individual possesses.

Even before entering school, much mathematics acquisition, just as language acquisition, has taken place for all normal children. For L2 students knowledge of mathematical facts, concepts, and algorithms is dependent on the quality of prior schooling. Students from Asian countries, for example, not infrequently have knowledge of mathematics far in advance of American peers. As one sixth-grader from Taiwan noted when asked to make comments about what is difficult and what is easy in doing mathematics in English, "all of it is very easy - except for the words."

**Heuristics**

Heuristics are the general mathematical strategies or 'rules of thumb'
used in problem solving.

One strategy L2 learners use is to draw on knowledge about algorithms without reference to linguistic knowledge. To illustrate, in a second grade class, no LEP child understood the worksheet headings **Vertical Addition** or **Three Addends** but all could do the addition task because presence of the symbol + signalled **add.** This type of strategy, of course, is not unique to LEP children at this age.

A discourse exchange between the teacher and a young ESL learner also illustrates strategic knowledge:

Teacher: (presenting a card showing 5 + 2)
What is the sum?
Student: 7
Teacher: What does **sum mean?**
Student: **a little bit**

Linguistically, the student did not understand the teacher's question but the symbol + signalled addition. Older learners show similar strategies. A class of 7 LEP students was given the problem, **Find the sum of the following numbers: 12, 15, 16.** All answered **13, 14.** They too did not know the term **sum** but drew on the linguistic clue, **following**, taking it to mean **subsequent** or **next.**

Juan, a third grade intermediate LEP student circled **4th of July, exploded,** and **firecrackers** as words he did not know in the problem:

> On the 4th of July 232 firecrackers exploded in the park. One hundred eighty firecrackers did not explode. What is the difference in the number of firecrackers?

Not understanding the problem, Juan's strategy was to by-pass the language, find numbers, and then simply subtract 4 from 232.

Mathematics learners at all levels develop strategies utilizing graphics or illustrations to solve problems without reference to linguistic knowledge. A grade 1 worksheet illustrates these sentences:

> There are 5 fish in all (with drawing of 5 fish).
> 2 are going away (drawing of 2 fish moving in the same direction).
> 3 are left (drawing of 3 fish remaining in the same place).

LEP students had no difficulty with subtraction. However, when the teacher questioned each child individually, she discovered that they did not know expressions **in all, going away, are left. In an addition problem showing 3 lions standing still and 2 lions running towards them, students answered** How many are there in all? **correctly but none understood the question. The strategy to rely on drawings and ignore verbal expressions in the problem worked.**

Use of graphic or symbolic cues is one part of metacognitive strategy available to L2 learners at all ages and proficiency levels. As language proficiency develops, increasing use of

linguistic strategies may appear. Strategies, however, do not necessarily assure success in mathematical performance. Preliminary findings from our pilot studies contrasting LEP students with native English-speaking peers at various age, grade, and proficiency levels suggest that the rate of failure in utilizing appropriate strategies is higher for LEP students.

**Control Processes**

Contral processes are metacognitive processes utilized in monitoring and evaluating mathematical performance. This regulation of cognition is essential in the complex activity of problem-solving.

Of particular relevance for the LEP student is the role of prior knowledge or experience. Knowledge of real world settings in which mathematical problem-solving occurs relates closely to the learner's culture. The socio-cultural context for problem-solving plays a critical role in regulatory strategies. If the learner has no familiarity with the content of the problem presented, if it requires experience beyond that offered through the learner's culture, mathematical problem-solving can be inhibited. A brief sampling of some data from our studies indicates the complexity of cross-cultural issues in mathematical problem solving.

A fourth grader was unable to solve a grocery store problem about buying a **box of Cheer**. Puzzled, he asked the teacher how **cheer** could come in a box when his only experience with **cheer** was **cheer at a soccer game**. Further discussion revealed that he had no experience with going to the supermarket. His mother did the grocery shopping while he was at school. Another student could not do a problem about scores in a game of bowling. He only knew **bowl** as a dish. With no knowledge of the sport, he had no solution for the problem.

Grade 5 students who had recently immigrated to the United States could not solve a series of problems dealing with U.S. currency when given graphic representations of coins. They simply did not know a quarter from a penny. Problems which displayed an array of coins followed by questions on how much was spent to buy various items called for either addition or subtraction, mathematical processes within the students' prior experience. However, these students had no experience at that point with the monetary system of their new surroundings. As a result, they could not determine **costs** and **money left** called for in a series of mathematical tasks.

Problems presented are often outside the language learner's life experiences. A high school junior brought the following problem to his ESL teacher for help. He was very good in mathematics but the problem in an Algebra II text was bewildering:

> Suppose you go into business raising thoroughbreds and quarter horses. Having studied linear programming, you decide to maximize the feasible profit you can make. Let x be the number of thoroughbreds, and let y be the number of quarter horses you raise each year. Write inequalities expressing each of the following requirements...Draw a graph of the feasible region... (Foerster 1984: 93)

**Thoroughbreds, quarter horses, linear programming,** and **maximizing feasible profit** presented notions far beyond the experience of this student.

Failure to understand the problem-solving context inhibits analysis of the problem information along with other aspects of regulatory behavior: organizing and carrying out the plan, checking results.

**Belief Systems**

Belief systems characterize an individual's view of mathematics and mathematical problem solving. As a result of experiences in doing mathematics, students develop a set of attitudes and beliefs that shape future performance. Affective aspects of problem solving are a part of the belief system and play an important role in mathematical performance in much the same way that we observe for language acquisition. Research in mathematics education indicates that self confidence in learning mathematics, low mathematics anxiety, and strong motivation to work on mathematics tasks have a significant, positive relationship with mathematics achievement. (See Reyes 1984 for a review of the literature.)

The graduate student who wrote that "I had full confidence in my ability to solve the mathematic problem" had a strong positive view about her ability to do mathematics. Our preliminary research findings show, in fact, an overall positive set of beliefs about mathematical calculations for the LEP students we have studied. Negative attitudes revolve around the language needed to solve problems.

## IMPLICATIONS FOR INTEGRATING MATHEMATICS AND LANGUAGE TEACHING

The processes of language acquisition and mathematics problem solving share many features cognitively and affectively. Close examination of the relationships between development of mathematical thinking and a second language indicates several key issues for instructional settings.

Mathematics instruction which provides alternate ways for getting input through small group work and peer tutor interactions can contribute, under appropriate conditions, to language development. Interaction in heterogeneous groups in which students learn to use one another as resources regardless of linguistic differences facilitates conceptual learning for all students. De Avila (1983) has shown that under classroom organizational conditions where LEP students are provided with access to interactive, cooperative learning situations, students can acquire both mathematics and English simultaneously. As part of the interaction, students use manipulative materials, graphics of various types, or other aids that make the activity meaningful. Materials such as these used in an interactive organizational framework give focus to the meaning of activites. Students comprehend the input, in other words. And they like doing the activities.

Literacy is crucial to the development of mathematical reasoning. Research in mathematics education shows that allowing students

opportunities to generate their own problems can contribute to successful mathematical performance (Silver & Thompson 1984). For LEP children lacking reading and writing skills, experience in writing and solving their own mathematics problems can contribute to successful language acquisition and mathematical performance.

As mathematics educators have found, personalizing problem statements, lowering anxiety levels, presenting situations that are related to life experiences and of practical significance to the learner have positive effects on performance (Silver & Thompson 1984). These are conditions which faciitate second language acquisition as well.

Linking mathematics and second language acquisition must be recognized for the complexities it presents. The cognitive and metacognitive aspects of mathematical reasoning must enter into any assessment of the role that mathematical input plays in facilitating L2 development. Affective variables are critical. The acquisition of language and mathematics can occur simultaneously if those elements which link both processes are taken into account. In large measure, the central link is language itself.

# BIBLIOGRAPHY

Burton, Leone. (1984). Mathematical thinking: The struggle for meaning. Journal for Research in Mathematics Education, 15, 35-49.

Canale, Michael. (1983). From communicative competence to communicative language pedagogy. In Language and communication, J.C. Richards & R. W. Schmidt, eds. New York: Longman.

Celce-Murcia, M. & Larsen-Freeman, D. (1983). The grammar book. Rowley, MA: Newbury House.

Crandall, J., Dale, T.C., Rhodes, N.C., & Spanos, G.A. (1985). The language of mathematics: The English barrier. To appear in Proceedings of the Delaware symposium on language studies VII, A. Labarca, ed. Norwood, NJ: Ablex.

Cuevas, Gilbert J. (1984) Mathematics learning in English as a second language. Journal for Research in Mathematics Education, 15, 134-144.

Cummins, Jim. (1984). Bilingualism and special education: Issues in assessment and pedagogy. San Diego, CA: College-Hill Press.

Dale, Theresa C. & Cuevas, Gilbert J. (1987). Integrating language and mathematics learning. In ESL through content-area instruction, JoAnn Crandall, ed. Englewood Cliffs, NJ: Prentice-Hall.

Dawe, Lloyd. (1984). A theoretical framework for the study of the effects of bilingualism on mathematics teaching and learning. Paper presented at the Fifth International Congress on Mathematical Education, Adelaide, Australia.

Dawe, Lloyd. (1983). Bilingualism and mathematical reasoning in English as a second language. Educational Studies in Mathematics 14.325-353.

Foerster, Paul A. (1984). Algebra and trigonometry functions and applications, revised edition. Menlo Park, CA: Addison-Wesley.

Halliday, M.A.K. (1975). Some aspects of sociolinguistics. In Interactions between linguistics and mathematics education: Final report of the symposium sponsored by UNESCO, CEDO and ICMI, Nairobi, Kenya, September 1-11, 1974. UNESCO Reort No. ED-74/Conf. 808. Paris: UNESCO, 64-73.

Hayes, C.W., Kessler, C., Bahruth, R. To read you must write: children in language acquisition. In Children and languages: Research, practice, and rationale for the early grades, K. E. Muller (ed.). New York: National Council on Foreign Language and International Studies. (in press).

Hiebert, James. 1984. Children's mathematics learning: The struggle to link form and understanding. The Elementary School Journal 84, 497-513.

Kessler, C., Quinn, M.E., & Hayes, C.W. (1985). Processing mathematics in a second language: Problems for LEP children. To appear in Proceedings of the Delaware symposium on language studies VII, A. Labarca, ed. Norwood, NJ: Ablex. ERIC Document ED 268 821.

Krashen, Stephen D. (1985) Inquiries and insights. Hayward, CA: Alemany Press.

Mayer, Richard E.(1986) Mathematics. In Cognition and instruction, R.F. Dillon & R.J. Sternberg, eds. Orlando, FL: Academic Press.

Mohan, Barnard A. (1986). Language and content. Reading, MA: Addison-Wesley.

Reyes, Laurie H. (1984) Affective variables and mathematics education. The Elementary School Journal, 84, 558-581.

Richards, Jack C. (1985). The context of language teaching. Cambridge: Cambridge University Press.

Saville-Troike, Muriel. (1984). What really matters in second language learning for academic achievement. TESOL Quarterly, 18, 199-220.

Schoenfeld, Alan H. (1985) **Mathematical problem solving**. Orlando, FL: Academic Press.

Searle, J. (1976). The classification of illocutionary acts. Language in Society, 5, 1-24.

Silver, E. A. (1987) Review of **Mathematical Problem Solving. Journal for Research in Mathematics Education**, 18, 53-58.

Silver, E.A. & Thompson, A.G. (1984) Research perspectives on problem solving in elementary school mathematics. The Elementary School Journal, 84, 529-545.